Title: Identification of damaging loads on the base of local strain measurements

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ABSTRACT

Methodology for identification of load's development (including non-linear effects due to local damages), causing measured structural deformations is presented. The intelligent sensor system monitoring load scenario and the corresponding damage (plastic yield, buckling and/or brittle crack) development can play the role of a *black box* allowing an after-accident-diagnosis..

INTRODUCTION

The objective is to present a methodology of damaging impact load identification on the base of local strain measurements done in few, properly chosen locations. The motivation to undertake the above research problem can be stimulated by the following examples of potential applications:

- load monitoring as the input to adaptive (in real time) impact energy dissipation systems and to design of structures exposed to dynamic loadings
- post-crash analysis allowing identification of cause of collision
- development of numerical tools supporting design process of impact energy absorbing systems,

The problem of identification of impact localisation for elastic membrane using the concept of so-called *smart layer* (thin layer with piezo-esnsors imbedded) has been discussed in [1,2,8,9]. The problem discussed in this paper is formulated more generally, including physically non-linear structural behaviour (e.g. plastic yielding).
Let us assume that sensors system (e.g. piezo-transducers) distributed on the structure is able to measure and store the history of local strains development.

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Then, the method of the corresponding load identification (the inverse problem) is based on so called *Virtual Distortion Method (VDM)* [3,4] making use of the *dynamic influence matrix Dij(t)*. This matrix describes the dynamic structural response (strain development in time) in observed locations *j* caused by unit impulse (in time *t=0*) applied in location *i* . Taking into account unknown load intensities in locations *i* and in time instances $\tau<t$, the objective function *F* describing mean-square *distance* between measured and modelled strains (in locations *j* and in time instances *t*) can be formulated. The problem of load identification leads in this case to the minimisation of the function *F* and can be based on the gradient optimisation technique, as the proposed approach allows numerically efficient, analytical sensitivity computation. The described above approach can be applied in the case of elastic structural response. In the case of elasto-plastic structural response, the analysis has to be generalised introducing additionally plastic distortions into the model (determined also on the base of the VDM method). Similar approach has been applied in structural adaptation to impact loads [6,7].
The problem formulation (restricted to small deformations) and methodology of the solution illustrated with numerical examples will be presented

VDM BASED DYNAMIC ANALYSIS OF ELASTO-PLASTIC SYSTEMS

In this chapter we will formulate the VDM based description of the dynamic response of elasto-plastic truss structure applying so-called *impulse-influence-matrix* $D_{ij}(t)$. The strain and stress development in element *i* of the structure loaded in nodes *n* and plastified in elements *k* can be expressed as follows:.

$$\begin{aligned}
\varepsilon_i(t) &= \sum_{\tau\le t}\sum_{n} D_{in}(t-\tau)\alpha_n^0(\tau) + \sum_{\tau\le t}\sum_{k} D_{ik}(t-\tau)\beta_k^0(\tau) \\
\sigma_i(t) &= E_i\left[\sum_{\tau\le t}\sum_{n} D_{in}(t-\tau)\alpha_n^0(\tau) + \sum_{\tau\le t}\sum_{k}\left(D_{ik}(t-\tau)-\delta_{ik}\right)\beta_k^0(\tau)\right]
\end{aligned} \qquad (1)$$

$\alpha_j^0(\tau)$ denotes in the above relations load intensities (components of external load) in node *j*, $\beta_k^0(\tau)$ denotes plastic distortion generated in element *k* and matrices $D_{in}(t-\tau)$ is called the impulse (or dynamic) influence matrices, describing dynamical response (strain development in time) in element *i*, caused by unit force applied in node *n*, and by unit virtual distortion modelling plastic yield in element *k* (in time instances $\tau \le t$). The unit force generating the influence matrix due to external loads is modelled via the Dirac's type of impulse. The elements $D_{ij}(t)$ can be determined by integration of the equations of movement (e.g. through the Newmark's algorithm), calculating deformations for the sequence of unit excitations of distortions in nodes *n* and elements *k*, while these indices run through nodes potentially loaded and elements potentially plastified. The influence matrices collect all information about the structure, including the boundary conditions.

Let us describe elasto-plastic physical conditions through the following piece-wise-linear formula with hardening (Fig.1):

$$\sigma_i(t)-\sigma_i^* = \gamma_i E_i\left(\varepsilon_i(t)-\varepsilon_i^*\right) \tag{2}$$

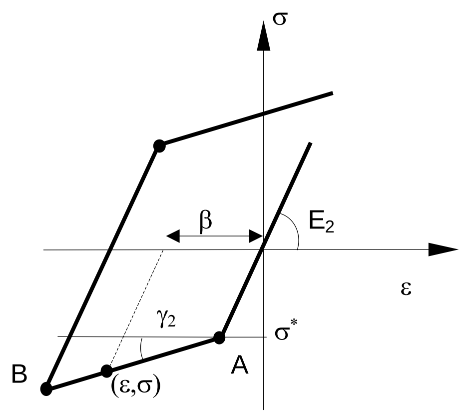

Figure. 1 Piece-wise-linear constitutive relation.

Substituting (1) to (2) and rearranging the formula, one can get the following equation:

$$\sum_{\tau\le t}\sum_{k}\left(\left(1-\gamma_i\right)D_{ik}\left(t-\tau\right)-\delta_{ik}\right)\cdot\beta_k^0\left(\tau\right)+ \\ +\sum_{\tau\le t}\sum_{n}\left(1-\gamma_i\right)D_{nj}\left(t-\tau\right)\cdot\alpha_n^0\left(\tau\right)=\left(1-\gamma_i\right)\varepsilon_i^* \tag{3}$$

Let us assume now, that the components of the influence matrices vanishes for the time instance $t=\tau$, cf. (4), what is true for sufficiently dense time step. Then, the influence of impulse generated in element *i* is seen in other elements not immediately, but in the subsequent time steps. In the consequence, determination (on the base of (3)) of the plastic zone development in the time step *t* is much simpler (cf.(5)):

$$D_{ij}\left(0\right)=0 \tag{4}$$

$$\beta_i^0\left(t\right)=-\left(1-\gamma_i\right)\varepsilon_i^*+\sum_{\tau<t}\sum_{n}\left(1-\gamma_i\right)D_{nj}\left(t-\tau\right)\cdot\alpha_n^0\left(\tau\right) \\ +\sum_{\tau<t}\sum_{k}\left(\left(1-\gamma_i\right)D_{ik}\left(t-\tau\right)-\delta_{ik}\right)\cdot\beta_k^0\left(\tau\right) \tag{5}$$

where the right hand side of equation does not depend on the time instance $t=\tau$. It depends only on the history of load and plastic zone development. The following relations have been used to reach the relation (5).

$$\sum_{\tau\le t}\sum_{n}(1-\gamma_i)D_{nj}(t-\tau)\cdot\alpha_n^0(\tau)=$$
$$=\sum_{\tau<t}\sum_{n}(1-\gamma_i)D_{nj}(t-\tau)\cdot\alpha_n^0(\tau)+\sum_{n}(1-\gamma_i)D_{nj}(0)\cdot\alpha_n^0(t)$$
$$\sum_{\tau\le t}\sum_{k}\left((1-\gamma_i)D_{ik}(t-\tau)-\delta_{ik}\right)\cdot\beta_k^0(\tau)= \tag{6}$$
$$=\sum_{\tau<t}\sum_{k}\left((1-\gamma_i)D_{ik}(t-\tau)-\delta_{ik}\right)\cdot\beta_k^0(\tau)+\sum_{k}\left((1-\gamma_i)D_{ik}(0)-\delta_{ik}\right)\cdot\beta_k^0(t)$$

If external load is determined ($\alpha_n^0(t)$ is given), then for each time step, after reaching the yield stress level in one structural element, we have to determine from the formula (5) the plastic zone development and use the plastic distortions determined in this way to calculation of the right-hand sides of equation (5) in the subsequent time steps.

PROBLEM FORMULATION AND GRADIENT-BASED METHOD OF THE IMPACT LOAD IDENTIFICATION

Let us assume that the external load is unknown, what means that $\alpha_n^0(t)$, describing load intensity development, together with plastic distortions, become the unknowns to be determined. Strain development measured in chosen elements are our input data allowing solving of the problem.
The objective function f can be defined as the mean-square *distance* between the measured and modeled responses of strain development (in localizations m and time instances t). Then, the load identification leads to the following process of the objective function minimization:

$$d_m(t)=\varepsilon_m^M(t)-\varepsilon_m(t) \tag{7}$$

$$f=\sum_{t}\sum_{m}\left[d_m(t)\right]^2 \tag{8}$$

where gradient based optimization techniques can be applied, as non-expensive analytical sensitivity analysis will be available.
First, let us consider a fully elastic case, without plastic distortions generated due to local overloading. Deformations $\varepsilon_m(t)$ are expressed by (9) in this case and gradient of the objective function takes the following form (10):

$$\varepsilon_m(t)=\sum_{\tau\le t}\sum_{n}D_{mn}^P(t-\tau)\alpha_n^0(\tau) \tag{9}$$
$$\frac{df}{d\alpha_n(\tau)}=-2\sum_{t}\sum_{m}d_m(t)D_{mn}(t-\tau) \tag{10}$$

where elements m are observed (equipped with sensors) and indices n run through potentially loaded nodes.

In the second case (elasto-plastic structure) the additional term responsible for plastic distortions has to be added to equation (1)a and the corresponding gradient of the objective function takes the form (10) where component $\frac{d\beta_k^0(\tau)}{d\alpha_n(\tau)}$ has to be determined from (2.3) by differentiating it with respect to α_n.

$$\frac{df}{d\alpha_n(\tau)} = -2\sum_t\sum_m d_m(t)\left(D_{mn}^P(t-\tau) + \sum_{\tau\le t}\sum_k D_{ik}^H(t-\tau)\frac{d\beta_k^0(\tau)}{d\alpha_n(\tau)}\right) \tag{10}$$

After differentiation, we can get (11) and making use of the formula (4), the relation (12) can be derived.

$$\begin{aligned}\sum_{\tau\le t}\sum_k\left((1-\gamma_i)D_{ik}(t-\tau)-\delta_{ik}\right)\cdot d\frac{\beta_k^0(\tau)}{d\alpha_n^0(\tau)} + \\ +\sum_{\tau\le t}\sum_n (1-\gamma_i)D_{nj}(t-\tau)\cdot\alpha_n^0(\tau) = (1-\gamma_i)\varepsilon_i^*\end{aligned} \tag{11}$$

$$\begin{aligned}\frac{d\beta_k^0(t)}{d\alpha_n^0(t)} = -(1-\gamma_i)\varepsilon_i^* + \sum_{\tau<t}\sum_n(1-\gamma_i)D_{nj}(t-\tau)\cdot\alpha_n^0(\tau) \\ +\sum_{\tau<t}\sum_k\left((1-\gamma_i)D_{ik}(t-\tau)-\delta_{ik}\right)\cdot\frac{d\beta_k^0(\tau)}{d\alpha_n^0(\tau)}\end{aligned} \tag{12}$$

Having gradient calculated, updating of $\alpha_n^0(t)$ can be performed in such a way that the objective function will be minimized. A simple, corresponding algorithm can be proposed as follows:

$$\alpha^0(t) = \alpha^0(t) + \nabla f(t)\cdot\delta \tag{13}$$

where: $\nabla f(t)$ - denotes the gradient of the objective function.

After updating of the load intensities new, dynamic, elasto-plastic analysis has to be performed, determining actualized development of plastic zones. Then, after actualized sensitivity analysis new updating of load intensities can be done. In this way, the iterative optimization process will lead us to the minimized objective function determining meantime the searched load distribution and the associated plastic distortions.

NUMERICAL EXAMPLE

Let us illustrate the proposed methodology with the example of truss beam structure shown in

Fig.2. assuming, for simplicity of demonstration that the structure is not reaching the yield stress level during loading process.
Dimensions are shown in Fig.2, while other data are specified below:
material density: 7800 $\left[{}^{kg}\!/\!{}_{m^3}\right]$, Young's modulus 2.1e11 [Pa], cross-sections of elements, equal for all elements: 1e-4 $\left[m^2\right]$

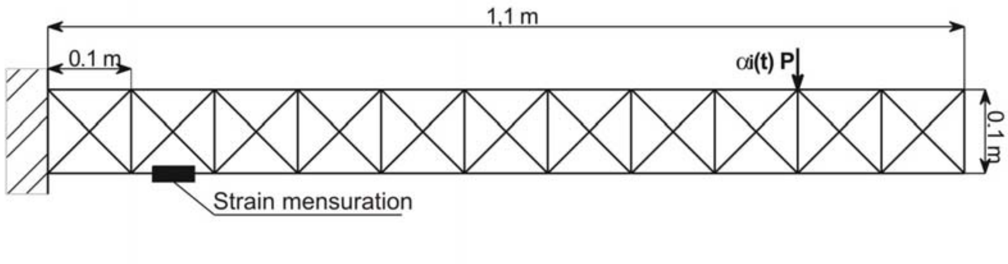

Fig. 2 Example of testing truss-beam structure

External load is modeled with the function shown in Fig. 3. A section of sinusoid starting in time step 50 (length of the time step: 0.05 ms), is ending in the time step 300.

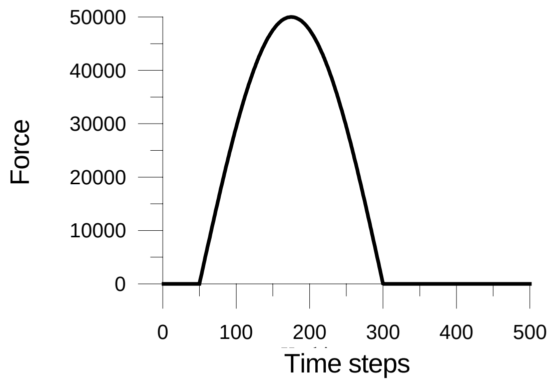

Fig. 3 Force modeling of load as the function of time steps.

Strains measured in time by the sensor (Fig.2) in response to load (Fig.3) applied in the node shown in Fig.2 is shown as the continuous line in Fig.4. Taking this line as the measured reference signal, the load identification procedure described above has been executed. The obtained result of this inverse problem is shown as the dotted line in Fig.4. We can see, that both lines are almost identical. The evolution of the objective function (3,2) is shown in Fig.5.

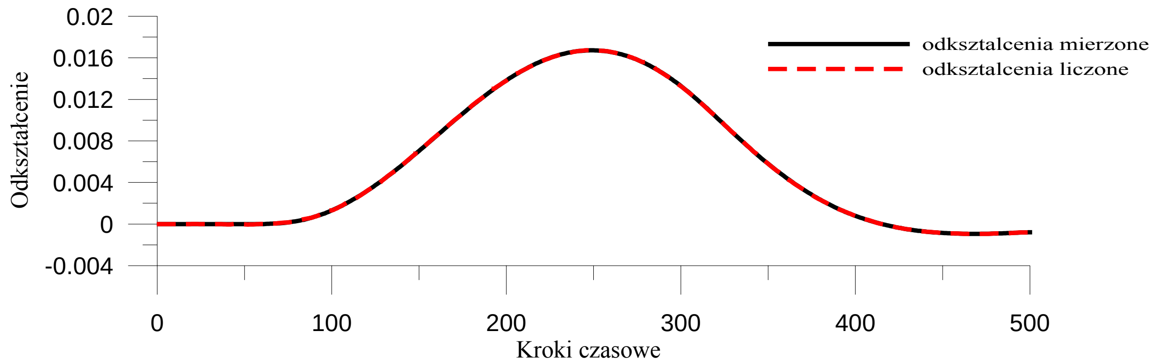

Fig. 4 „Measured" strains versus modeled strains

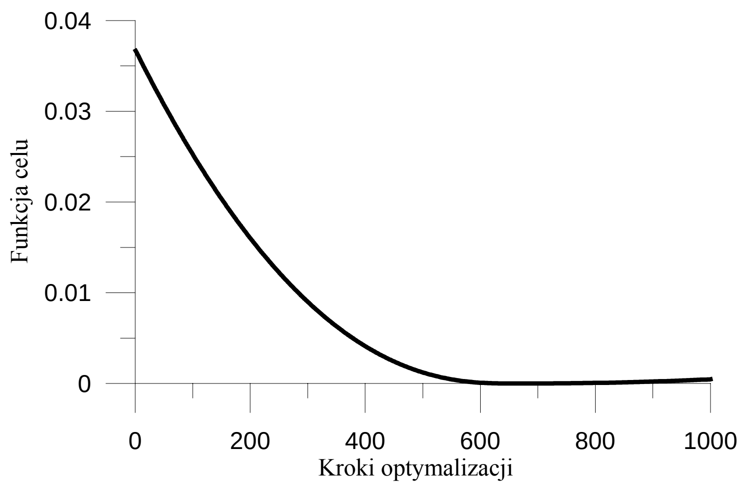

Fig. 5 The objective function evolution

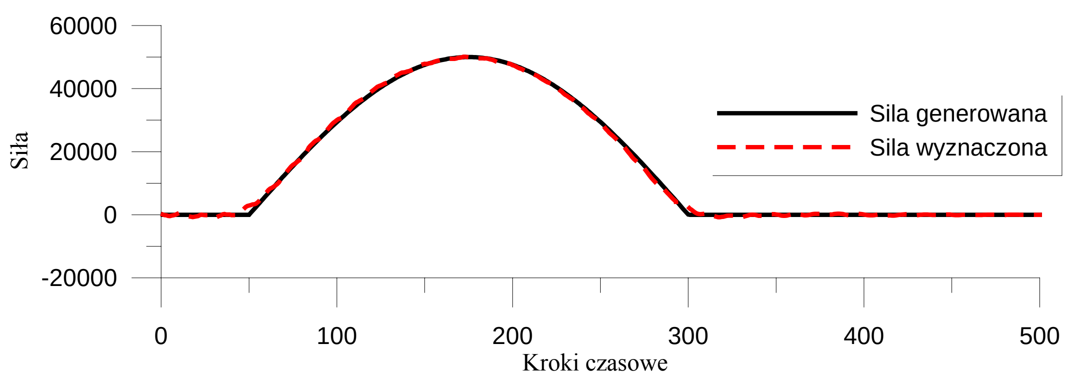

Fig. 6 Applied load versus identified load

The „applied" external excitation (cf.Fig.3) versus identified loads are shown in Fig,6. Almost perfect match of both lines can be observed.

CONCLUSIONS

The methodology of identification of the load history (and the corresponding plastic zone development) in structures equipped with sensors measuring strains in few chosen elements is proposed. In many cases just one sensor observing local strain development in time allows precise identification of loading conditions.
The proposed approach makes use of gradient-based optimization technique (minimization of the objective function denoting *distance* between measured and modeled signals). Analytical sensitivity analysis and automatic structural remodeling (VDM) do not require actualization of the global stiffness matrix and repetition of calculation of dynamic structural responses, what reduces significantly the numerical effort.
It is possible to apply the proposed concept to develop automatic systems (accident black boxes) allowing diagnosis (made *aposteriori*) concerning determination of environmental conditions causing emergency situation.
The proposed method of load identification can be also used in complex intelligent systems responding adaptively to variable environmental conditions (e.g. active adaptation to measured and identified in real time impact conditions).

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